Modeling the subglacial hydrology of the James Lobe of the Laurentide Ice Sheet

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Abstract

To investigate the drainage conditions that might be expected to develop beneath soft-bedded ice sheets, we modeled the subglacial hydrology of the James Lobe of the Laurentide Ice Sheet from Hudson Bay to the Missouri River. Simulations suggest the James Lobe had little effect on regional groundwater flow because the poorly conductive Upper-Cretaceous shale that occupies the upper layer of the bedrock would have functioned as a regional aquitard. This implies that general northward groundwater flow out of the Williston Basin has likely persisted throughout the Quaternary. Moreover, the simulations indicate that the regional aquifer system could not have drained even the minimum amount of basal meltwater that might have been produced from at the glacier bed. Therefore, excess drainage must have occurred by some sort of channelized drainage network at the ice–till interface. Using a regional groundwater model to determine the hydraulic conductivity for an equivalent porous medium in a 1-m thick zone between the ice and underlying sediment, and assuming conduit dimensions from previous theoretical work, we use a theoretical karst aquifer analog as a heuristic approach to estimate the spacing of subglacial conduits that would have been required at the ice–till interface to evacuate the minimum water flux. Results suggest that for conduits assumed to be on the order of a tenth of a meter deep and up to a meter wide, inter-conduit spacing must be on the order of tens–hundreds of meters apart to maintain basal water pressures below the ice overburden pressure while evacuating the hypothesized minimum meltwater flux.

1. Introduction

The rapid fluctuations of the James, Des Moines, Green Bay and Lake Michigan Lobes draining the southern margin of the Laurentide Ice Sheet (LIS) (Fig. 1) (Clayton and Moran, 1982; Mickelson et al., 1983) under relatively low driving stresses (Mathews, 1974; Clark, 1992; Hooyer and Iverson, 2002) require a significant component of ice motion by basal sliding and/or subglacial sediment deformation (Alley, 1991; Clark, 1994; Hooyer and Iverson, 2002). These processes of bed dynamics have broader implications for interpretations of global and abrupt climate change where rapid changes in ice sheet dimensions may have influenced the 100-ka ice age cycle, caused the discharge of large quantities of icebergs to the ocean and affected continental runoff with attendant consequences for ocean circulation (see review in Clark et al., 1999). The occurrence and importance of basal sliding and/or subglacial sediment deformation to ice sheet motion are linked to the subglacial hydrology of the ice sheet. Of particular importance is the ability of the regional aquifer to drain meltwater (Alley, 1989), and the ability of whatever basal drainage system that might develop to discharge excess meltwater that cannot be accommodated by the regional aquifer.

The absence of modern analogs to the mid-latitude, terrestrial paleo-ice lobes of the LIS and the limited access to the ice streams of the West Antarctic Ice Sheet, which are the most closely related modern analogs (e.g. Patterson, 1998), have necessitated the use of numerical model simulations to investigate the subglacial hydrology of ice sheets and lobes. Previous modeling studies have found that subglacial aquifers were unable to drain meltwater at
the ice–till interface without the development of a basal drainage system such as a laminar film (Alley et al., 1989; Breemer et al., 2002), a channelized system (Brown et al., 1987; Walder and Fowler, 1994; Ng, 2000), or periodic outburst floods (Piotrowski, 1997a, b) to maintain stable ice flow. However, as Breemer et al. (2002) recognized, because films are unstable (Walder, 1986; Engelhardt and Kamb, 1997), they would most likely be only an idealized hydrological equivalent of a channelized network such as the system of shallow subglacial conduits predicted from the theoretical work of Walder and Fowler (1994) and Ng (2000). Detailed modeling of a system of such conduits, predicted to be on the order of only meters wide but distributed over a region on the order of 100s of kilometers wide, is beyond the state of the art for current models and computing power. Accounting for their overall effects is still crucial, however, because although such conduits may occupy only a small fraction of the aquifer volume or even of the total porosity, they would be the dominant mechanism for regional flow. This dilemma is familiar to karst modelers, however.

A well-known approach in modeling karst systems is to treat the aquifer as an equivalent porous medium (EPM), in which “the relevant physical properties (e.g., frequency, diameter, and tortuosity of tubes for capillary tube models; frequency and aperture width for parallel-fracture models) are combined so that discharge per unit hydraulic gradient is equivalent to that produced by Darcy’s Law” (Vacher and Mylroie, 2002). The EPM approach has obvious limitations. First, it can only characterize aquifers in terms of general, large-scale properties, most notably, regional hydraulic conductivity. Second, there is some uncertainty associated with the fact that flow within conduits is generally turbulent flow, while the application of Darcy’s Law inherently assumes laminar flow. Inferring specific characteristics such as the types and dimensions of porosity from the general properties of an EPM, such as hydraulic conductivity or effective porosity, is therefore problematic. Nevertheless, such heuristic approaches can be useful for placing quantitative boundaries on unknown and unobservable properties, as long as the limitations of the approach are explicitly acknowledged (cf., Scanlon et al., 2003).

Here we simulate the interaction between the James Lobe of the LIS (Fig. 1) and the regional aquifer system extending from the Hudson Bay Lowland south to the terminus of the James Lobe in South Dakota. Based on known characteristics of the regional aquifer and hypothetical minimum subglacial meltwater production rates, we find that, as in other studies, a subglacial drainage network would have been necessary to prevent ice flotation. We then use an EPM model, in combination with other previous theoretical work, to infer the range of effective porosity, in terms of conduit spacing or “conduit density” of a drainage network that could have maintained basal water pressures near the ice load while evacuating a minimum meltwater flux. Until more rigorous models are available, this heuristic approach offers a useful first approximation of the properties a basal drainage system at the ice–till interface for an ice lobe.

2. Model configuration

2.1. Boundary conditions and parameters

The James Lobe was the westernmost of the distinctive lobes that drained the southern margin of the LIS. Ice flow feeding the lobe originated in the Hudson Bay Lowland (Fig. 1). Clark (1992) calculated that the lobe had a driving stress of $\sim 1.0$ kPa. It reached its maximum extent in South Dakota $\sim 20^{14}$C ka BP with subsequent margin fluctuations before retreating at $\sim 14^{14}$C ka BP (Clayton and Moran, 1982).
To model the subglacial hydrology of the James Lobe, we run steady state and transient simulations with the three-dimensional, finite-difference groundwater modeling program MODFLOW (McDonald and Harbaugh, 1988). The model domain extends from Hudson Bay to the Missouri River (Fig. 1). The eastern boundary is the Red River. The western boundary is the Manitoba–Saskatchewan border. The northern boundary corresponds with the ice divide at 14 14C ka BP reconstructed by Dyke and Prest (1987). The other boundaries are arbitrarily chosen at sufficient distance from the lobe that conditions can be assumed to reflect the ambient state of the regional aquifer, beyond the influence of the lobe. The model domain is underlain by Paleozoic sedimentary bedrock in the Hudson Bay Lowland, Precambrian crystalline bedrock of the Canadian Shield, and Paleozoic and Mesozoic sedimentary rocks of the North American plains (Fig. 2). The rock layers thicken and dip towards the north in the Hudson Bay Lowland and toward the southwest under the plains into the Williston Basin.

We group the rock layers into aquifers and aquitards following Downey (1986) and Downey and Dinwiddie (1988), and assign isotropic hydraulic conductivities \( K \) \( [LT^{-1}] \) (based on order-of-magnitude regional averages) of Lennox (1993) (see Table 1). As we demonstrate below, this configuration produces hydraulic heads and groundwater flow patterns in good agreement with the modern hydrology of the modeled region (Downey, 1986; Downey and Dinwiddie, 1988). A layer of till that thickens, from 1 to 15 m, towards the south is added atop these layers. We use the term *till* in the broad sense to describe Quaternary sediment that is actually a combination of basal till, supraglacial debris flows, glaciofluvial sand and gravel, and glaciolacustrine sediment. These types of glacial units have \( K \) values that range over 8 orders of magnitude (Stephenson et al., 1988). We thus group them into a sediment hydrostratigraphic unit assigned a \( K \) of \( 3 \times 10^{-7} \text{ m s}^{-1} \), which is at the upper end of unweathered basal till, the average of weathered basal till, and the lower end of glaciofluvial sediment (Stephenson et al., 1988). To test the sensitivity of the results to this assumption, we vary \( K \) between \( 3 \times 10^{-8} \) and \( 3 \times 10^{-6} \text{ m s}^{-1} \). Above the till layer, we add an artificial layer by which to apply the water-equivalent constant hydraulic head of the lobe. In the model, we treat the contact with the Precambrian igneous and metamorphic rocks as a no-flow boundary because the \( K \) values of these rocks are orders of magnitude lower than the \( K \) values of the aquifer units (Breemer et al., 2002). Grid size is 10 km \( \times \) 10 km. In transient runs, a specific storage value of \( 4.6 \times 10^{-6} \text{ m}^{-1} \) is assigned to all layers (Breemer et al., 2002).

Constant hydraulic head values are assigned to the boundary of each layer from reconstructions of the predevelopment potentiometric surface (Hitchon, 1969a, b; Downey, 1986; Downey and Dinwiddie, 1988). For the western boundary, these reflect recharge from the Rocky Mountains, where the rock layers are exposed at the surface. For the steady-state control run simulating modern conditions, the water table is held constant at the surface elevation (the top of the upper-most model layer). Recharge of 0.2 m yr\(^{-1}\) is applied to the upper-most model layer to simulate precipitation minus evaporation (data from NOAA-CIRES Climate Diagnostics Center). In the full glacial simulations, constant hydraulic head values equivalent to the load inferred from the 14 14C ka BP maximum ice sheet of Licciardi et al. (1998) are imposed on the upper-most layer in the model above the till layer. Recharge of 0.006 m yr\(^{-1}\) is input to the till layer to simulate the contribution of the basal geothermal melting of the ice sheet (Drewry, 1986; Paterson, 1994). This value is thought to be typical for geothermal melting under most ice sheets and provides a minimum value for basal meltwater flux. We exclude surface meltwater penetrating to the glacier bed (e.g. Zwally et al., 2002) and meltwater
from frictional heating in these simulations. Such sources of meltwater may play an important role in subglacial hydrology and ice sheet motion, but the objective of our simulations is to provide bounds for the type of basal hydraulic system that would be required to discharge even the minimum volume of water that could be produced by the ice sheet. Including the additional sources of meltwater would imply that the actual system would have to be even more robust than our estimated minimum system. We also exclude the effect of permafrost at the boundary, which would presumably make for less efficient drainage (e.g. Breemer et al., 2002). The hypothetical system thus provides a basis for bounding the hydraulic conditions that would be associated with the least demanding basal water flux and the least restrictive discharge conditions at the southern boundary.

To validate the model, we simulated steady-state regional groundwater flow for modern boundary conditions and parameters (Fig. 3a) (Table 2). We then stimulated a hypothetical steady state for glacial conditions. (Fig. 3b) (Table 2). For reasons discussed below, it is highly unlikely that steady-state conditions were ever achieved, but modeling steady state provides a necessary benchmark for understanding and evaluating simulations of transient conditions. Two transient runs simulating evolution of conditions to 2500 and 10,000 years duration were performed to evaluate the characteristic time of response of the aquifer system beneath the James Lobe (Fig. 4) (Table 2). The 2500-year simulation estimates the impact that the James Lobe may have had during its longest occupancy of its maximum extent (Clayton and Moran, 1982). The 10,000-year simulation provides an estimate of conditions approaching steady state. These simulations are initiated with the steady-state modern control heads. The water-equivalent hydraulic head of the James Lobe is added to the top layer of the model, and the model is run for 2500 and 10,000 simulated years at 50-year time steps.

Table 2

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2.2. Subglacial drainage system models

The dimensions of subglacial drainage conduits in soft sediment are not yet well known. From theoretical considerations, Walder and Fowler (1994) estimated a height on the order of 0.1 m for broad, shallow “canals” in sandy/silty till and up to 1.0 m in gravel beneath ice sheets. Subsequent numerical modeling by Ng (2000) supported the prediction of their analytical model. We thus take Walder and Fowler’s value of 0.1 m as the minimum for the thickness of the till in the conduit drainage zone. On the other hand, Carsey et al. (2002) reported a saturated space of about 1.4 m height beneath Kamb Ice Stream (Ice Stream C). For our calculations, we thus assumed minimum conduit heights on the order of 0.1 m, bounding the contribution of conduit height to effective porosity within an order of magnitude.

Given the low hydraulic conductivity of glacial till, a conduit system in the till layer will dominate by many orders of magnitude the total conductivity of the aquifer it permeates. Porous flow in till lying beneath the zone occupied by the conduits will not contribute significantly to the total conductivity; till lying below the conduit network, no matter how thick, can thus be regarded as impermeable with respect to flow in the direction of the conduits. The thickness of the aquifer zone in which effective porosity of the subglacial aquifer is to be measured is thus the thickness of the till layer in which the conduits are contained. Effective porosity in such a zone is thus a function of conduit volume and density (e.g. spacing). Because we assumed conduits on the order of 0.1 m height within a layer 1 m thick, our estimates of effective porosity constitute minimum values within an order of magnitude.

To constrain the characteristics of a basal drainage system at the glacier bed, we ran a suite of simulations of groundwater flow in a 1-m thick aquifer between the ice and underlying till, adjusting the hydraulic conductivity until discharge was just sufficient to accommodate the specified (minimum) basal meltwater flux with basal water pressure kept at or just below the ice overburden pressure (Table 2). With these hydraulic conductivities, we constrain the properties of conduits in the aquifer unit by applying theoretical conduit dimensions (e.g. Walder and Fowler, 1994; Engelhardt and Kamb, 1997; Ng, 2000) to an EPM model (Vacher and Mylroie, 2002) to evaluate the relationship between equivalent bulk hydraulic conductivity and the diameter and density of a system of ideal parallel,

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1Walder and Fowler (1994) used the term “canal” to distinguish subglacial conduits of broad, shallow cross-section beneath soft-bedded ice from the semi-circular Rothlisberger and Nye “channels” beneath hard-bedded ice. Although the distinctions between these terms are understood in glacial geology and glaciology, we have adopted the terms “conduit” or “tube” as they are applied in engineering and hydrogeology to describe phreatic caves carrying “closed” flow in lieu of the terms “canal” or “channel”, which are associated with open-channel flow in disciplines outside of glacial science. Walder and Fowler (1994) and Ng (2000) used the term “depth” for the vertical height of the conduit. We adopt the term “height” in lieu of “depth” because “depth” is more generally associated with water depth in open-channel flow, whereas conduit dimensions are more precisely described in terms of radius for circular conduits, or height and width for non-circular conduits.
circular tubes in a subglacial environment (Fig. 5) (Appendix A).

Since the theoretical work to date (Walder and Fowler, 1994; Engelhardt and Kamb, 1997; Ng, 2000) suggests that subglacial conduits on soft beds are likely to be “flattened” compared to their counterparts on hard-beds, it is necessary to determine the hydraulic equivalence of such conduits to the circular cross-section conduits assumed in the Vacher–Mylroie EPM model. For purposes of this analysis, we assume, following Ng (2000), an elliptical cross-section. For any non-circular conduit, the diameter of a hydraulically equivalent circular conduit (i.e., one that would carry the same volume discharge under a given hydraulic gradient) is estimated as 4 times the hydraulic radius of the non-circular conduit (in either laminar or turbulent flow for a given roughness) (cf., McCabe et al., 2005, pp. 101, 120). Fig. 6 shows the equivalent diameters for conduits of elliptical cross-section with heights (2b, where b is the semi-minor axis) of 0.10, 0.20 and 0.30 m, and widths (2a, where a is the semi-major axis) ranging from 0.2 to 3.0 m (Appendix B). Note that the eccentricity of the conduit cross-section extracts a high price in hydraulic efficiency, which decreases abruptly for cross-sections with aspect ratios greater than ~2. For conduits of depth 0.10 m, the equivalent diameter is constrained at ~0.16 m, regardless of the increase in width after a width of ~0.5 m. A conduit depth of 0.20 m has an equivalent diameter of ~0.31 m. For conduits of 0.30 m depth and 1.0 m width, the effective diameter is less than half the major axis. The notable observation from this analysis is that elliptical subglacial conduits with heights over the range suggested by theoretical work to date are unable to achieve hydraulically equivalent diameters of more than ~0.15–0.45 m. Circularity of the conduit is clearly an important factor in promoting efficient drainage; thus a soft-bedded ice sheet can evacuate water more efficiently with a larger number of smaller but more circular conduits than with fewer conduits of larger cross-sectional area but higher aspect ratios. This would seem to favor conduits of relatively modest aspect-ratio, perhaps 2–3, consistent with the dimensions suggested from previous theoretical work (Ng, 2000). We note that although the actual conduits beneath ice sheets are likely to form an anastomosing network, a system of parallel tubes probably provides a suitable approximation because anastomosing conduits are essentially sub-parallel (i.e. Clark and Walder, 1994; Ng, 2000). We test the sensitivity of the simulated basal drainage system to the \( K \) of the till layer below the drainage system by varying \( K \) from 3 \( \times 10^{-8} \) and 3 \( \times 10^{-6} \) m s\(^{-1}\). We note that estimates of hydraulic conductivity for a given volume discharge and hydraulic gradient bear a simple inverse relationship to aquifer thickness (Appendix A). We also note that, as in karst aquifers, although conduits may contribute only a small
fraction of the total volume or total porosity, they are highly conductive and constitute the effective porosity for regional flow with the low-conductivity matrix accommodating a small to insignificant amount of flow. Flow within the matrix follows local gradients toward the conduits and does not contribute significantly to the regional flow pattern.

The EPM approach, of course, assumes Darcy’s Law, which is based on laminar flow. However, comparisons between EPM models and field observations in karst aquifers suggest that equivalency models can accurately simulate groundwater flow in karst regions (cf., Scanlon et al., 2003). Moreover, we note that because the effect of turbulence is to make flow less efficient, the consequence of using a model that omits it is that an even more robust drainage system would be required than one estimated from the model. Because our objective is to find the bounds for the most conservative system required, such results therefore suffice. More rigorous simulations would require the use of complex, “hybrid” models that couple a continuum, laminar model and a turbulent, pipe network model (e.g. Liedl et al., 2003; Bakalowicz, 2005). As noted earlier, the scale of the James Lobe system precludes such an approach, even if a suitable hybrid model could be developed. With these qualifications in mind, we thus apply the equivalent porous media model of Vacher and Mylroie (2002) to estimate the plausible orders of magnitude for conduit spacing.

These inferences for the subglacial conduit system are evaluated against two additional strategies for estimating conduit spacing. In the first, we solve an empirical relationship between conduit spacing, ice thickness and melt rate modified from Carlson (2004) using varied ice thicknesses, \( z_i \) [\( L \)], and melt rates, \( m_r \) [\( LT^{-1} \)], integrated over 1 year [\( T \)] in which

\[
x = \sqrt{\left( \frac{\left(\frac{\gamma_i z_i}{\gamma_w} - x\right) m_r^{-1}}{2} - 1 \right)},
\]

where \( x \) [\( L \)] is the half-distance between canals, \( \gamma_i \) is the unit weight of ice [\( ML^{-3} \)], and \( \gamma_w \) [\( ML^{-3} \)] is the unit weight of water. This equation determines the maximum distance (twice \( x \)) that can exist between two conduits (anastomosing or parallel) with the pore water pressure remaining below ice flotation level in the till. The dimensions of the conduit (width and depth) are neither included nor required in this calculation. We solve this equation for melt rates, \( m_r \), between 0.005 and 1.0 m yr\(^{-1}\) and consider those encompassing the range estimated for glaciers of 0.006–0.1 m yr\(^{-1}\) (Paterson, 1994). This range includes the effects of frictional heating, with the upper end even consistent with the addition of surface meltwater to the ice sheet bed (e.g. Zwally et al., 2002). We use ice thicknesses, \( z_i \), between 0 and 1500 m and consider thicknesses between 200 and 500, covering the range of ice thicknesses for the James Lobe and other ice lobes of the southern LIS (Mathews, 1974; Clark, 1992; Licciardi et al., 1998).

For the second additional method, we set up a series of high-resolution, small-domain (i.e., meter-scale cells within a 100-m-scale domain) hypothetical ice sheet drainage configurations with conduits represented by drains in
MODFLOW. We increased the spacing between conduits under two uniform 200 and 500-m water-equivalent thick ice sheets until pore water pressure exceeded ice overburden. The ice sheet is applied in the same manner as in the full-scale simulations of the James Lobe. The conduits are configured in square patterns with a maximum conduits spacing being the diagonal across a given square. Grid cell size is 1 m × 1 m. The till is 3 m thick and has a $K$ of $3 \times 10^{-7}$ m s$^{-1}$. This configuration is an attempt at approximating the anastomosing configuration of canal drainage systems (Clark and Walder, 1994; Ng, 2000). These conduits are 1 m wide, with 10-cm heights. Recharge of 0.006 m yr$^{-1}$ is applied to the till layer. We also test the drainage system response by applying a range of ice sheet surface gradients applicable to the ice lobes of the southern LIS (Mathews, 1974; Clark, 1992; Licciardi et al., 1998).

3. Results

3.1. Regional aquifer response

To assess the skill of our model, we compare our control simulation against the observed hydraulic head in aquifers of the western North American plains reported in Downey (1986) and Downey and Dinwiddie (1988) (example locations denoted on Fig. 3a by the black dots with 1-$\sigma$ variability indicated by error bars around the black dots). The steady-state control simulation of modern conditions is in agreement with other simulations and observations (Hitchon, 1969a, b; Downey, 1986; Downey and Dinwiddie, 1988; Lennox, 1993). In the C–O aquifer (see Fig. 2a), our simulated hydraulic head decreases from 950 m in the southwest to 400 m in the northeast. This closely matches the observed hydraulic head, which decreases from 940 to 395 m. In the M aquifer, hydraulic head is 700 m in the southwest decreasing to 400 m in the northeast, close to the observed hydraulic heads of 730 m in the southwest to 450 m in the northeast. In the L–K aquifer, our modeled hydraulic head decreases from 420 m in the southwest to 300 m in the northeast, matching well with the observed hydraulic head of 450 m in the southwest decreasing to 300 m in the northeast. In the control simulation, groundwater flow in the till is northeastward, toward Hudson Bay. The time to steady state as predicted by the Fourier number (Carslaw and Jaeger, 1959) varies between units, with aquifers such as the L–K sandstone taking only $\sim$200 years to reach steady state whereas the deeper C–O aquifer takes $\sim$11,000 years to reach steady state. Aquitards, with the exception of the U–K, take much longer to reach steady state, on the order of 100,000 years. The exception, U–K, takes only 2300 years to reach steady state.

In this steady-state glacial simulation, pore water pressure at the ice–till interface, as indicated by the pressure difference between the hydraulic head of the James Lobe (water-equivalent) and the hydraulic head in the till layer, is in excess of ice-overburden pressure (90% of the ice thickness), which would cause ice flotation. Varying till $K$ between $3 \times 10^{-8}$ and $3 \times 10^{-6}$ m s$^{-1}$ has little effect on pore water pressure, and pore water pressure in the till remains greater than ice overburden.

In the transient simulations (Fig. 4), groundwater flow in the till and U–K layer is southward but the Lower Mesozoic and Paleozoic layers in the Williston Basin maintain a component of northeastward flow. The ice lobe only affects groundwater flow in these lower layers near where they are exposed at the surface between the edge of the U–K aquitard and the Canadian Shield (Fig. 2). In the southwestern area of the model, hydraulic head values in the Paleozoic layers remain near present values and groundwater flow direction is similar to the steady-state control simulation of modern conditions. For both simulations pore water pressure at the ice–till interface exceeds ice overburden pressure.

3.2. Subglacial drainage system simulations

Treating the 1-m thick zone between the “ice” (the upper-most model layer) and the underlying till as an EPM, we increased $K$ until the basal water pressure dropped just below ice overburden pressure. Increasing $K$ to $\sim$30 m s$^{-1}$ reduced pore water pressure to slightly above ice-overburden pressure for the entire James Lobe (Fig. 7). At $\sim$300 m s$^{-1}$, basal pore water pressure is still above ice flotation except near the terminus, where it is slightly below flotation. As $K$ is increased to $\sim$3000 m s$^{-1}$, there is an abrupt drop in till pore water pressure, and the basal pore water pressure across the entire lobe becomes less than half of the ice-overburden pressure, indicating that $K$ for an EPM that could drain the excess meltwater lies between $\sim$300 and $\sim$3000 m s$^{-1}$. We note that the inclusion of this simulated drainage system in the model does not alter the bedrock aquifer flow pattern, and bedrock groundwater flow remains reversed similar to the steady-state simulation without a drainage system.

By plotting the intersection between the range of $K$ values obtained for the 1-m-thick EPM from our simulations and the range of diameters of circular cylindrical tubes that are hydraulically equivalent to elliptical tubes of the James Lobe near to 2300 m in the C–O in Hudson Bay. The time to steady state, as predicted by the Fourier number (Carslaw and Jaeger, 1959) varies between units, with aquifers such as the L–K sandstone taking only $\sim$200 years to reach steady state whereas the deeper C–O aquifer takes $\sim$11,000 years to reach steady state. Aquitards, with the exception of the U–K, take much longer to reach steady state, on the order of 100,000 years. The exception, U–K, takes only 2300 years to reach steady state.
height and widths suggested from the work of Walder and Fowler (1994) and Ng (2000), we identify the associated ranges of effective porosity and tube density for the EPM (Appendix A). In Fig. 5, we show the bounds (gray area) for tube density, \( N/A \), and effective porosity, \( n \), for \( K \) between 300 and 3000 m s\(^{-1}\), with conduit height of 0.10–0.30 m and width from 0.5 to 1 m. For a height of 0.1 m, conduits are spaced between 0.6 m apart when \( K = 3000 \) m s\(^{-1}\), and 6 m apart when \( K = 300 \) m s\(^{-1}\), with effective porosity of 0.5 and 0.05%, respectively. Considering a height of 0.20 m, spacing increases to between 4 m (\( K = 3000 \) m s\(^{-1}\), effective porosity = 0.2) and 40 m (\( K = 300 \) m s\(^{-1}\), effective porosity = 0.02). At a height of 0.30 m, spacing is between 20 m (\( K = 3000 \) m s\(^{-1}\), effective porosity = 0.09) and 200 m (\( K = 300 \) m s\(^{-1}\), effective porosity = 0.009) apart. Results are insensitive to the \( K \) used for the underlying till (i.e. \( 3 \times 10^{-8} \) and \( 3 \times 10^{-6} \) m s\(^{-1}\)). Thus the spacing estimates are sensitive to the assumed height of the conduit, with deeper conduits being more efficient and allowing increased spacing between conduits.

It has long been established in sediment transport theory and practice that stream channels in erodable materials will tend to maintain an equilibrium water velocity and channel cross-section (cf. Mackin, 1948). Indeed, the central hypothesis underlying Walder and Fowler’s (1994) theoretical model of subglacial channels in soft sediment was that the dimensions of the bottom half of the conduit must be determined by the equilibrium between sediment erosion on the one hand, and the inward creep and deposition of sediment on the other, in addition to the equilibrium between inward creep and melting of the ice in the upper half. Accordingly, turbulent flow is required to erode sediment and maintain equilibrium conduit shape.

To determine whether the discharge predicted by our EPM model is consistent with turbulent flow through conduits of the dimensions predicted by Walder and Fowler (1994) and Ng (2000), we calculate velocity and Reynolds number from the Darcy–Weisbach equation for circular tubes with equivalent diameters, \( D_{eq} \), equal to the limiting equivalent diameters, \( LD_{eq} \) (see Fig. 6 and Table 3) for elliptical tubes of 0.10, 0.20 and 0.30 m height with hypothesized surface irregularity relief, \( e \), of 0.3 mm for the smoothest case and 3.0 mm for the roughest case (Table 3 and Appendix B). For these calculations, we apply the water flux produced by a basal melt rate of 0.006 m yr\(^{-1}\) and the regional hydraulic gradient from our modeled transect (\( dh/dx = 9.82 \times 10^{-4} \), for \( dh = 2160 \) m, \( dx = 2200 \) km). For the range in heights and roughness, spacing is between \( \sim 10 \) and 280 m with velocity between \( \sim 0.24 \) and 0.70 m s\(^{-1}\) (see Table 3). Flow is turbulent in these conduits, and the spacings between conduits are consistent with the EPM results.

There are no empirical data on subglacial conduit erosion, but the classic work on canal hydraulics by Fortier and Scobey (1926) identifies “maximum permissible
velocities’ for canals dug into various earth materials (Chow, 1959). These range from 0.46 m s\(^{-1}\) for clear water running over fine sand to 1.52 m s\(^{-1}\) for water transporting non-colloidal silts, sands, gravels or rock fragments over coarse gravel. Beyond these velocities, channel scour exceeds deposition, so equilibrium, at least for alluvial channels should be at velocities approaching these values. The velocities cited above for turbulent flow from the Darcy–Weisbach formula fall within this range.

From Eq. (1) (Carlson, 2004) (Fig. 8), higher melt rates and thinner ice require more closely spaced conduit. For the range of melt rates (0.006–0.1 m yr\(^{-1}\)) and ice

Table 3

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<th>Reynolds number</th>
<th>Conduit spacing</th>
<th>Mean velocity in elliptical conduit, aspect ratio = 3</th>
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<td>0.15 m</td>
<td>0.004 m(^3) s(^{-1})</td>
<td>0.24 m s(^{-1})</td>
<td>3.7 (\times) 10(^{5})</td>
<td>10 m</td>
<td>0.17 m s(^{-1})</td>
</tr>
<tr>
<td>0.2 m</td>
<td>0.31 m</td>
<td>0.006 m(^3) s(^{-1})</td>
<td>0.35 m s(^{-1})</td>
<td>5.3 (\times) 10(^{5})</td>
<td>15 m</td>
<td>0.25 m s(^{-1})</td>
</tr>
<tr>
<td>0.3 m</td>
<td>0.46 m</td>
<td>0.030 m(^3) s(^{-1})</td>
<td>0.40 m s(^{-1})</td>
<td>1.25 (\times) 10(^{6})</td>
<td>100 m</td>
<td>0.32 m s(^{-1})</td>
</tr>
<tr>
<td>0.46 m</td>
<td>0.086 m(^3) s(^{-1})</td>
<td>0.55 m s(^{-1})</td>
<td>1.73 (\times) 10(^{6})</td>
<td>210 m</td>
<td>0.45 m s(^{-1})</td>
<td></td>
</tr>
<tr>
<td>0.70 m s(^{-1})</td>
<td>2.42 (\times) 10(^{6})</td>
<td>0.46 m s(^{-1})</td>
<td>3.28 (\times) 10(^{6})</td>
<td>280 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.52 m s(^{-1})</td>
<td>3.28 (\times) 10(^{6})</td>
<td>0.55 m s(^{-1})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8. Graph of the solutions to Eq. (1) (Carlson, 2004) with varied melt rate and ice thickness. Arrows show the maximum and minimum half space (half the distance between conduit) for the range of melt rates and ice thicknesses considered.
thicknesses (200–500 m) considered here, the pore water pressure at the ice sheet bed will remain below flotation with conduits spaced every 50–320 m (Fig. 8). Our second additional method, the high-resolution, small-scale hypothetical ice sheet basal drainage model simulations, indicates that there is a rapid increase in pore water pressure to the ice flotation level when conduit spacing is increased from 42 to 45 m (Fig. 9). This suggests a maximum conduit spacing of ~42 m if the ice sheet is to remain coupled to its bed (Fig. 9a). These results are insensitive to the range of ice thicknesses used (i.e. 200–500 m water-equivalent head) and to the addition of an ice sheet surface gradient. Comparisons of the hydraulic heads from simulations without an ice sheet surface gradient indicate that local, matrix groundwater flow in the till is directed towards the nearest conduit, with the ice sheet surface gradient having an insignificant effect on matrix groundwater flow direction.

4. Discussion

Although the steady-state glacial simulation predicts that groundwater flow in the western glaciated plains would be reversed were the ice sheet geometry to persist for longer than our longest transient run (10,000 years), the transient runs are in fact more consistent with the history of the James Lobe, which occupied its maximum position for less than 1000 years (Clayton and Moran, 1982). Prior to ~23 $^{14}$C ka BP, the southwestern margin of the LIS was restricted to the Precambrian Canadian Shield (Dyke et al., 2002), and thus did not directly influence groundwater flow in the North American plains. Ice subsequently advanced off the shield at ~22 $^{14}$C ka BP (Dyke et al., 2002) and reached its maximum position ~20 $^{14}$C ka BP, followed by several large fluctuations before beginning general retreat after 14 $^{14}$C ka BP (Clayton and Moran, 1982).

Our transient 10,000-year simulation thus provides an upper bound for the impact of the James Lobe on the groundwater flow of the North American plains. This simulation suggests that the James Lobe had little effect on regional groundwater flow below the Quaternary, U–K and L–K layers, with the exception of the C–O layer, which would be nearing steady state after 10,000 years. However, the impact of the James Lobe on the C–O aquifer was likely even less than suggested by the 10,000-year simulation because the James Lobe never occupied its maximum extent for more than 1000 years (Clayton and Moran, 1982). Thus, flow in the lower aquifers and aquitards was likely to have remained northeastward due to the low K of the overlying U–K aquitard (Fig. 4).

These results contradict the qualitative inferences by Downey (1986) and Downey and Dinwiddie (1988) that these aquifers would experience a reversal of flow direction during glaciations. However, our model results are in close agreement with the isotopic spring data of Grasby and Chen (2005), which indicate reversal of groundwater flow in the aquifers only near surface exposures that were in direct contact with the James Lobe. Thus, if James Lobe advances prior to the Last Glacial Maximum were also short-lived, then northeastward groundwater flow under the North American plains is likely to have been unperturbed.

Similar modeling studies of other lobes of the LIS and the Scandinavian Ice Sheet (SIS) have shown that results are sensitive to differences in regional hydrogeological conditions. Breemer et al. (2002) demonstrated a complete reversal of groundwater flow within 2900 years under the Lake Michigan Lobe of the LIS. Likewise, transient and steady-state modeling under the SIS show complete reorganization of groundwater flow in response to glaciation.
(Boulton et al., 1993, 1995; Piotrowski, 1997a, b). These regions lack a regional aquitard such as the U–K shale, which hydrologically isolated the lower layers beneath the James Lobe. Accordingly, we conclude that groundwater flow in glaciated regions underlain by more permeable bedrock like that of the Great Lakes region of North America was likely significantly influenced by the ice lobes whereas regions underlain by less permeable substrates such as the North American plains were not.

Both our steady state and transient simulations suggest subglacial aquifers under the James Lobe could not have drained the base of the lobe rapidly enough to maintain basal pore water pressure below the ice flotation level. This requires meltwater drainage at the ice–till interface via a film (e.g. Alley, 1989) or a conduit system (e.g. Walder and Fowler, 1994; Ng, 2000) to prevent flotation. Our results suggest that a subglacial drainage system at the ice–till interface composed of conduits with widths of 0.5–1.0 m and heights of 0.1 m–0.3 m (Walder and Fowler, 1994; Engelhardt and Kamb, 1997; Ng, 2000), would most likely have been spaced on the order of tens–hundreds of meters apart to have drained the excess meltwater under the James Lobe efficiently enough to keep the lobe coupled to its bed (Figs. 5–7). We acknowledge the uncertainties in inferring actual drainage system geometry from EPM parameters, but note that this approach has the simple and modest objective of identifying the order-of-magnitude scale of a system that could plausibly evacuate the minimum water flux from beneath an ice lobe. More rigorous and precise calculations must await the development of numerical models and computers that can incorporate integrated matrix and conduit flow for large-scale systems.

Moreover, the EPM results are realistic and in good agreement with the results of our independent calculations for turbulent flow, the separate calculations from the two other models (Figs. 8–9), and with field data all indicating a conduit spacing on the order of 10s to 100s of meters. Fieldwork by Engelhardt and Kamb (1997), for example, suggested a conduit system under the Whillans Ice Stream, West Antarctica, with conduit dimensions consistent with those predicted by Walder and Fowler (1994) and spacing between 50 and 300 m. As noted earlier, our model omits melting due to basal frictional heating and surface meltwater that penetrates to the glacier bed. If these additional meltwater sources are included, then an even higher equivalent K (implying larger and/or more closely spaced conduits) is needed to keep pore water pressure below flotation in the karst analog model. We note that there is good agreement between our simulated effective pressures for southern Manitoba and preconsolidation test results on tills in southern Saskatchewan which indicate subglacial effective pressures between ~160 and 190 m water-equivalent (Sauer et al., 1993).

Evidence of conduit drainage systems in the geologic record is somewhat sparse because the sediment deposited by the conduits may have been reworked into the till by subsequent shearing and deformation. Clark and Walder (1994) presented a summary of the geologic evidence for conduit systems for the LIS, including the region covered by the James Lobe, and suggested that sand lenses in till are the remnants of conduit drainage systems. Clayton et al. (1989) noted that sand lenses are common occurrences in till (including the till deposited by the James Lobe) and concluded that they were deposited by subglacial water flowing in conduits at the ice–till interface. Further north near the origin of the James Lobe in the Hudson Bay Lowland, Carlson et al. (2004) documented similar sand lenses that they interpreted to be the remnants of subglacial conduits. In addition, we note that these sand lenses have characteristic widths on the order of 1 m and heights of on the order of 1–10 cm conduit.

Interestingly, our results from both the regional-scale EPM and high resolution–small domain simulations show an abrupt decrease in pore water pressure (Figs. 7 and 9) once a threshold in conduit spacing is crossed. We therefore hypothesize that conduit spacing might vary under an ice sheet, with pore water pressure remaining below ice flotation until a critical spacing is reached. Increasing conduit spacing beyond this threshold causes a rapid rise in pore water pressure because the till can no longer discharge the meltwater produced to the conduits, and the ice sheet begins to float. This is similar to what has been observed under small valley glaciers where the diurnal melt cycle decreases the coupling of the glacier sole and the underlying sediment during periods of high pore water pressure (Iverson et al., 1995). We further hypothesize that the incipient flotation of the ice sheet, however, would promote the formation of new conduits because as effective pressure approaches zero, sediment piping can occur. Conduit spacing would then decrease, with an attendant decrease in pore water pressure, causing the ice sheet to recouple with its bed. Thus, ice sheet basal drainage systems may be self-limiting systems. This suggests that if the James Lobe had a drainage system similar to that proposed for the Whillans Ice Stream, West Antarctica (Ice Stream B) (Engelhardt and Kamb, 1997), then this lobe could have remained coupled to its bed, rather than experiencing the widespread bed decoupling suggested for the neighboring Des Moines Lobe (Hooyer and Iverson, 2002) and the SIS (Piotrowski and Tulaczyk, 1999; Piotrowski et al., 2004).

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Appendix A

From the Kozeny–Carman equation (Bear, 1972; Scheidegger, 1974; Halek and Svek, 1979):

\[ K = \frac{npD^2}{32\mu x} \]

Vacher and Mylroie (2002, Eq. (9)), (A.1)

where \( K \) is the equivalent hydraulic conductivity \([LT^{-1}]\) of a porous medium with drainage canals, \( n \) is the effective porosity of the sediment layer \([L^3L^{-3}]\), \( \rho \) is fluid density \([ML^{-3}]\), \( g \) is gravity \([LT^{-2}]\), \( D \) is cylindrical tube diameter \([L]\), \( \mu \) is fluid viscosity \([ML^{-1}T^{-1}]\), and \( x \) is tube tortuosity \([L^{-1}]\) assumed to be 1. Tube diameter, \( D \), is then related to tube density by

\[ N = \frac{4n}{\pi D^2} \]

Vacher and Mylroie (2002, Eq. (13)), (A.2)

where \( N \) is the number of tubes in an area, \( A \ [L^2] \).

Appendix B

The classic Darcy-Weisbach formula

\[ h_L = \frac{LV^2}{2Dg} = \frac{8fLQ^2}{gD^5\pi^2} \]

(Hassted Methods et al., 2004, Eq. (2.17))

(A.3)

originally developed for pipe hydraulics, is also applied to the hydraulics of natural conduits in karst aquifers (cf., White, 1988, Eq. (6.20)). \( h_L \ [L] \) is the head loss over distance \( L \ [L] \), \( V \) is the mean fluid velocity \([LT^{-1}]\), \( D \) is cylindrical tube diameter \([L]\), \( g \) is gravity \([LT^{-2}]\), \( Q \) is discharge \([L^3T^{-1}]\), and \( f \) is the friction factor, which can be approximated by the Swanee–Jain formula

\[ f = \frac{1.325}{[\ln(e/3.7D + 5.74/Re^{0.9})]^2} \]

Haestad et al. (2004, Eq. (2.20))

(A.4)

where \( e \) is the surface irregularity relief \([L]\), and \( Re \) is the Reynolds number.

References


