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THE DOMINANCE OF DISPERSION IN THE EVOLUTION OF BED MATERIAL WAVES IN GRAVEL-BED RIVERS

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ABSTRACT

Bed material waves are temporary zones of sediment accumulation created by large sediment inputs. Recent theoretical, experimental and field studies examine factors influencing dispersion and translation of bed material waves in quasi-uniform, gravel-bed channels. Exchanges of sediment between a channel and its floodplain are neglected. Within these constraints, two factors influence relative rates of dispersion and translation: (1) interactions between wave topography, flow and bed load transport; and (2) particle-size differences between wave material and original bed material. Our results indicate that dispersion dominates the evolution of bed material waves in gravel-bed channels. Significant translation requires a low Froude number, which is uncharacteristic of gravel-bed channels, and low wave amplitude which, for a large wave, can be achieved only after substantial dispersion. Wave material of small particle size can promote translation, but it primarily increases bed load transport rate and thereby accelerates wave evolution. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: bed material waves; bed load; gravel-bed rivers

INTRODUCTION

Routing of sediment through channel systems from its sources, which commonly are punctuated in time and space, is key to understanding and predicting channel response to watershed disturbance. One approach to the sediment routing problem is to investigate bed material waves – transient zones of sediment accumulation in channels that are created by sediment inputs and do not owe their existence solely to variations in channel topography. Understanding how single waves disperse and propagate should improve sediment routing in a channel network (Benda and Dunne, 1997).

A basic issue of the behaviour of bed material waves and their effect on stream channels is their relative rates of translation and dispersion. The term 'wave' is adopted here as a general term to allow the possibility of a mass of bed material to propagate as a disturbance that translates as well as disperses, analogous to a water wave or seismic wave. A bed material wave that evolves only by dispersion is still regarded as a wave. We define a purely dispersive wave as one where the apex and trailing edge do not migrate downstream (Figure 1A). Depending on background transport rates, the backwater effect of the wave may cause significant deposition upstream and, if water is ponded, a delta may form (Figure 1B). Thus a dispersive wave can spread both upstream and downstream of its point of origin; its centre of mass may or may not shift downstream, depending on upstream deposition. In a natural channel, however, topography can force streamwise variations in deposition, and the precise location of the apex can be difficult to discern. A translational wave is one in which all features, including leading and trailing edges, wave apex and centre of mass, advance downstream (Figure 1C). A reasonable expectation is for a wave to exhibit both translation and dispersion (Figure 1D).

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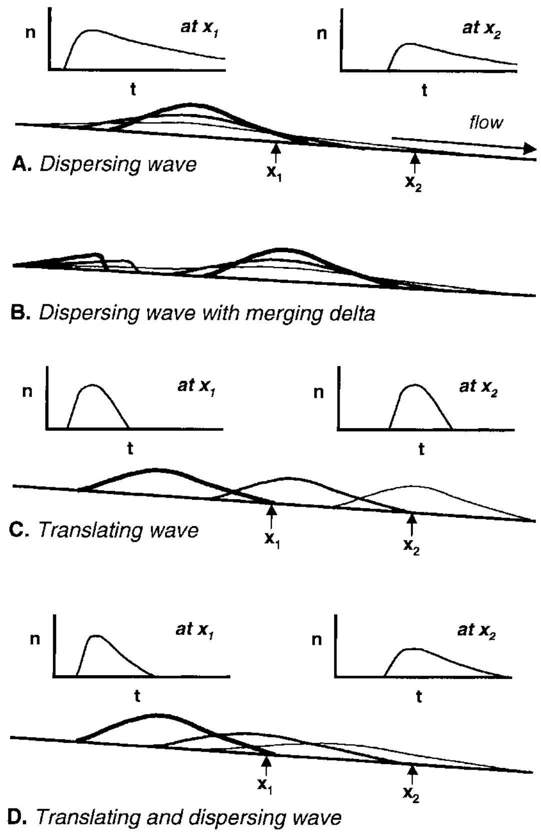

Figure 1. Idealized types of bed material waves in profile. The heaviest line represents the original wave profile; successively lighter lines represent stages in wave evolution; n, bed elevation; t, time; x, channel distance

These distinctions are important for river resources. Dispersion of sediment inputs attenuates but prolongs impacts downstream, whereas translation preserves pronounced sequences of impact and recovery.

A crude indication of relative rates of dispersion and translation is whether or not translation is observed unequivocally. Records of at-a-station changes in bed elevation can give ambiguous indications of whether an evolving sediment wave is dispersive (Figure 1A) or translational (Figure 1C). In either case, two or more stations record lagged sequences of aggradation followed by degradation, thus peaks in aggradation shift downstream with time. The difference is that, in a purely dispersive wave, streamwise deviations from pre-wave bed elevations always decrease downstream at each stage of evolution, whereas, in a translational wave, bed elevations recover to pre-wave elevations sequentially downstream. The distinction may be difficult to make, particularly if a wave is both dispersive and translational (Figure 1D) and pre-wave bed elevations are poorly known.

Expectations of wave behaviour probably have been influenced by Gilbert's (1917) landmark study of sediment waves produced by hydraulic mining of placer deposits along tributaries of the American and Sacramento Rivers in California. His conclusions regarding wave behaviour emphasize translation: 'The downstream movement of the great body of debris is thus analogous to the downstream movement of a great body of storm water, the apex of the flood traveling in the direction of the current.'(p. 30, col. 2) However, evidence supporting this model is equivocal. Sediment waves in the major tributaries 'flattened' with time, and peak levels of aggradation generally decreased downstream. Profiles of waves cannot be reconstructed in enough detail to detect translation. For example, variations of bed elevations at three sections including a tributary (Yuba River) and the Sacramento (Figure 4, Gilbert, 1917) resemble the pattern of Figure 1A: aggradation peaked later but was also less further downstream. Erosion of remaining large deposits of mining debris along Bear and American Rivers continues to sustain elevated sediment yields compared with conditions before mining, thus delaying the full recovery of bed elevations as predicted by Gilbert (James, 1989, 1993, 1997). To be fair, it must be said that Gilbert's purpose was not to understand the nature of sediment waves, but to measure and predict aggradation in a large natural system with many sediment sources and tributaries and having wide variations in transport capacity and competence. Other studies of sediment waves in rivers also are ambiguous in documenting significant translation (Roberts and Church, 1986; Knighton, 1989; Pitlick, 1993; Madej and Ozaki, 1996).

The purpose of this paper is to summarize recent theoretical models, experiments and field studies that have been designed specifically to measure relative rates of dispersion and translation of bed material waves. We address the question: What conditions favour translation or dispersion of bed material waves in uniform, gravel-bed channels? In considering uniform channels only, we neglect stream-wise variations in channel and valley topography that force deposition or erosion at the reach scale. We also disregard exchanges of sediment between a channel and its floodplain. Within this scope, two factors influence wave behaviour: (1) interactions between flow, wave topography and bed load transport; and (2) relative particle sizes of input sediment and pre-existing bed material.

INTERACTIONS OF FLOW AND WAVE TOPOGRAPHY

Theory

Mathematical models predicting the evolution of bed material waves are derived from one-dimensional expressions of the St Venant equations for shallow flow (conservation of fluid mass and momentum), the Exner equation (conservation of sediment), a friction equation relating the flow to tractive forces, and a bed load function (unspecified) relating tractive force to bed load transport.

$$\frac{1}{g}\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2g} + h + n\right) + \frac{\tau}{\rho g h} = 0 \tag{1}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = 0 \tag{2}$$

$$\frac{\partial n}{\partial t} + \frac{1}{1-p}\frac{\partial q_s}{\partial x} = 0 \tag{3}$$

$$\frac{u}{\sqrt{\tau/\rho}} = c_f \tag{4}$$

$$q_s = f\,(\tau, D) \tag{5}$$

where u and h are fluid velocity and depth, t and x are time and distance, n is bed elevation, τ is boundary shear stress, ρ is fluid density, g is gravitational acceleration, p is bed porosity, q_s is bed load transport rate, D is sediment particle size and c_f is a friction coefficient. Numerical models using these equations have been derived for aggradational profiles downstream of an increase in sediment supply, and similar approaches are borrowed for bed material waves, which include the upstream limb of an aggradational profile. These models are described fully elsewhere (Lisle *et al.*, 1997; Cui and Parker, in preparation; Dodd, 1998).

We can identify factors that influence dispersion and translation from an equation that can be derived from Equations 1–5 and applied to sediment-wave evolution, even without an analytical solution. Through substitutions and using the Meyer–Peter–Mueller (MPM) bed load equation and a constant friction coefficient, Dodd (1998) combines Equations 1–5 into a form of the Exner equation written in terms of the Froude number, $F = u/(gh)^{0.5}$ (see Appendix A):

$$\frac{\partial n}{\partial t} = \frac{Kqc_f^{1/2}}{R_s\,(1-p)}\left[\frac{\partial^2 n}{\partial x^2} + \left(\frac{\partial}{\partial x}\left(1-F^2\right)\frac{\partial h}{\partial x}\right) + \cdots\right] \tag{6}$$

where K is an empirical constant in the MPM equation, q is unit water discharge, R_s is the submerged specific gravity of sediment, and F is the Froude number. The unspecified terms in the brackets are unsteady flow terms, which are small for $F < 1$.

Equation 6 is meant only to portray some general aspects of wave behaviour and does not represent adequately all adjustments between flow, bed topography and sediment transport in natural gravel-bed rivers. Several simplifying assumptions other than one-dimensionality are implicit in the model. Water discharge is steady, and only bed load transport is considered. Effects of mixed particle sizes and the low exceedence of critical shear stress, which are typical of gravel-bed rivers, are not included. Other assumptions are given later.

However, Equation 6 does reveal some aspects of wave behaviour, as well as some difficulties entailed in numerical solutions. Obviously, the rate of wave evolution depends on bed load transport rate, which is expressed parametrically by the term outside the brackets. The first term within the brackets expresses the rate of wave dispersion: the bed aggrades in the concave-upward extremities of the wave $\left(\frac{\partial^2 n}{\partial x^2} \text{ positive}\right)$ and degrades in the convex-upward, central portion $\left(\frac{\partial^2 n}{\partial x^2} \text{ negative}\right)$. The second term expresses the rate of translation: If $F < 1$, then $\frac{\partial h}{\partial x}$ in the translation term is negative on the upstream limb of the wave (depth decreasing downstream) and positive on the downstream limb (depth increasing). This indicates scour of the upstream limb $\left(\frac{\partial n}{\partial t} \text{ negative}\right)$ and deposition on the downstream limb $\left(\frac{\partial n}{\partial t} \text{ positive}\right)$, thereby manifesting translation downstream. As F approaches 1, the translation term goes to zero, indicating that waves in flows approaching critical are predominantly dispersive. This is significant because steep, self-formed, gravel-bed channels tend to have high Froude numbers during high discharge (Wahl, 1993; Grant, 1997). If $F > 1$, the wave translates upstream.

Variations in F along a profile create problems for numerical solutions to Equation (6). If F is alternatively <1, >1, or 1 everywhere, then relatively simple numerical or analytical solutions can be derived but, as average F approaches 1, flow is likely to go critical somewhere on a wave profile. In these circumstances, the time-scale of bed disturbances approaches that of the flow, and the validity of the assumption of quasi-steady flow that justifies ignoring unsteady-flow terms and decoupling equations of the flow from those of the sediment is challenged. However, decoupled numerical solutions can still be applied to the wave problem because the wavelength of the perturbation is long.

Three numerical solutions predict wave evolution using different simplifying assumptions to reach a solution. Equation 13 in Pizzuto's model (Lisle *et al.*, 1997), which is equivalent to Equation 6 in this paper, is solved using step-backwater methods to determine the velocity and depth for steady, gradually varied flow. Dodd (1998) uses the McCormack finite-element approximation (Chaudry, 1987), which is a coupled solution assuming the existence of unsteady flow. This requires certain boundary conditions and is not used for non-uniform sediment.

The model of Cui and Parker (in preparation) incorporates more physical processes characteristic of natural gravel-bed rivers. Like Dodd (1998), they allow F to vary over the profile but they assume quasi-uniform flow over each channel element and apply a longer space step and time relaxation for $F \approx 1$. Their model incorporates the low exceedence of critical shear stress, effects of mixed bed material (armouring, selective transport and varying roughness), and particle abrasion. We used this model to predict evolution of bed material waves in our experiments and field studies. All three models predict strong dominance of dispersion over translation of waves in flows of high F, which are characteristic of gravel-bed channels (see Figure 8 in Lisle *et al.*, 1997), and further impedance of wave translation when the amplitude is high enough (roughly at the scale of the flow depth or greater) to create significant backwater effects.

Field and experimental evidence

These predictions are confirmed by two experiments and a field example. Lisle *et al.* (1997) introduced a bed material wave into an experimental channel (average $F = 0{\cdot}90$) carrying a relatively high sediment load. The wave dispersed symmetrically, although a delta such as depicted in Figure 1B was not observed, probably

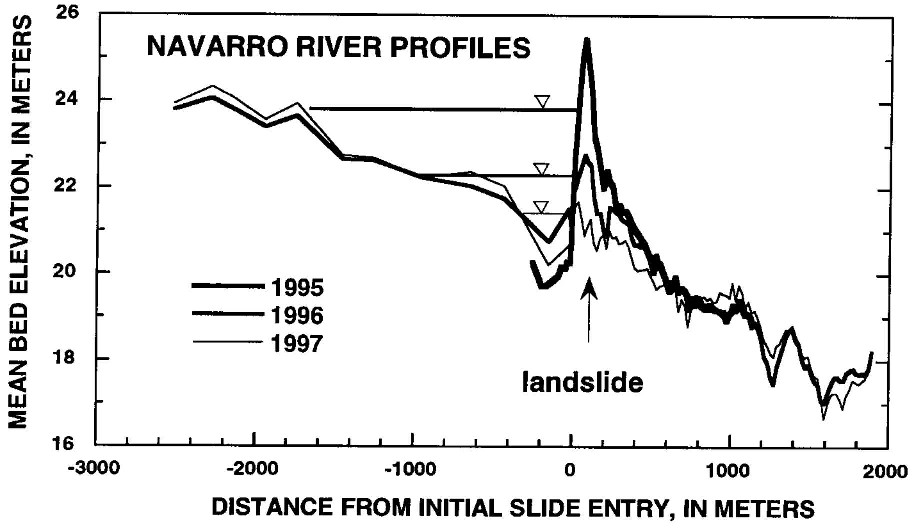

Figure 2. Longitudinal profiles of mean bed elevation of the Navarro River in the vicinity of a bed material wave created by a landslide in 1995. Horizontal lines depict water surface elevations of the pond upstream of the landslide

because the reservoir filled before changes in the bed profile were first measured. In a similar experiment, Cui *et al.* (in review a) introduced a gravel wave into a gravel-bed channel formed under low sediment feed rates (average $F = 0{\cdot}96$). The wave did not translate, and an upstream delta was observed. The evolution of the wave is predicted accurately by the model of Cui and Parker (in preparation; Cui *et al.*, in review b). In each of these experiments, transport of sediment in suspension was negligible.

A well-documented field example is provided by a bed material wave that originated from a landslide that entered the Navarro River in northern California in 1995 (Hansler *et al.*, 1998; Sutherland *et al.*, 1998) (Figure 2). Over the next 3 years, the sediment from the landslide spread as a thinning wedge downstream. Meanwhile, a backwater created by the landslide deposit caused a sediment wedge to form upstream and advance until it filled the ponded reservoir and merged with the eroding landslide deposit, thereby creating the profiles of Figure 1B. Each year, abrasion and release of fine material converted approximately 25% of the highly weathered landslide material to suspended sediment, which was transported out of the study reach. The fate of the suspended sediment was not measured, but apparently it contributed to the dispersion of the sediment wave. The evolving wave profile within the study reach was predicted accurately by the Cui-Parker model (Hansler *et al.*, 1998).

It is significant to note that in both of these examples, the flow was fully three-dimensional, allowing sediment and flow to interact with bar–pool channel topography, but the evolution of profiles of mean bed elevation were predicted accurately by one-dimensional models.

PARTICLE-SIZE EFFECTS

The analysis above indicates that bed material waves in gravel-bed channels are not inherently translatable owing to interactions of flow, bed load transport and wave topography. In order for a wave to translate downstream, the trailing limb, which has a gentler slope than the pre-existing bed, must be eroded. A reasonable scenario for this to occur is if the input sediment were finer and thus more mobile than the underlying bed material. Below, we review experiments, model predictions and field evidence for particle-size effects on wave behaviour in order of increasing contrast between wave- and bed material.

Mixed size waves

In the first case, we posit that the surface of a sediment wave having the same size composition as the bulk bed material would be more mobile than that of an unaffected reach because the increased sediment supply manifested in the wave would reduce armouring (Dietrich *et al.*, 1989). If ambient sediment transport rates were low enough to promote strong armouring elsewhere on the bed and to delay filling behind the wave (conditions that were not met in the experiment and field example described above), the trailing limb of the wave perhaps could be eroded and advance downstream. To test this, we formed a coarse surface layer in an experimental channel by feeding a mixture of equal parts of sand and gravel at a low rate on to a bed predominantly of gravel and, after equilibrium in sediment transport was achieved, introduced a sediment wave of the same sediment mixture as the feed (Cui *et al.*, in review a). The sand formed extensive smooth areas of bed over which the gravel was readily mobilized and transported (Iseya and Ikeda, 1987; Whiting *et al.*, 1988). This apparently accelerated the evolution of the wave, but the wave failed to translate (Figure 3A).

Sandy waves

In another run, a pure sand wave was introduced into the channel with the same bed material (Cui *et al.*, in review a). Smooth bed areas were formed more extensively than during the previous run, and the wave advanced more rapidly still. The trailing edge also advanced, at first more slowly than the leading edge

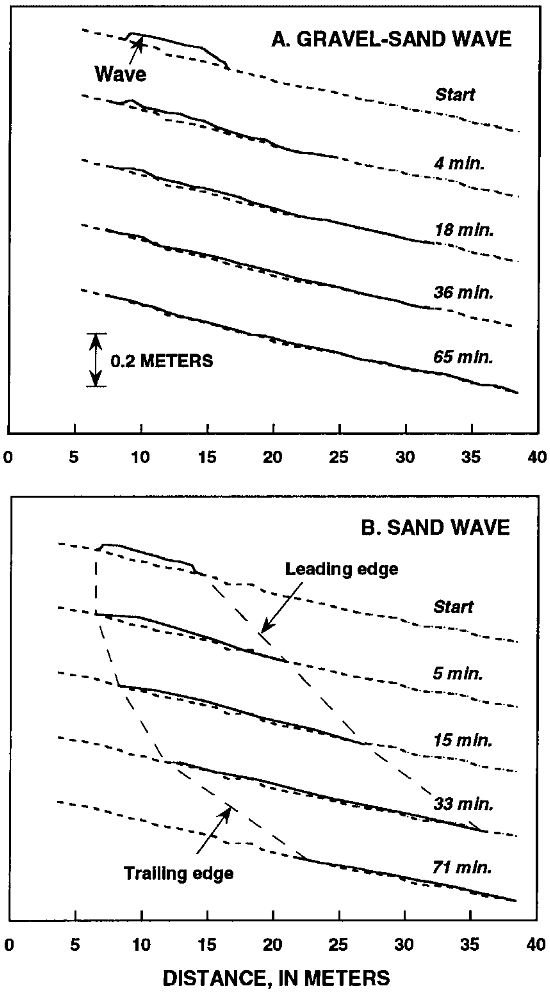

Figure 3. Profiles of stages of evolution of a bed material wave in an experimental channel (Cui *et al.*, in review a): (A) run 2, gravel–sand wave; (B) run 4B, sand wave. The dashed line represents the original profile and the solid line represents the profile at successive cumulative run times in minutes

because of backwater effects of the wave but, as wave amplitude was reduced, the trailing edge accelerated to nearly the celerity of the leading edge (Figure 3B). Therefore, the wave translated. However, the evolution of the wave profile would be better characterized as dispersion than translation, because the length of the wave more than doubled during the experiment and the amplitude of the wave was greatly reduced.

An approximate natural example of sandy waves in gravel-bed channels is provided by the North Fork of the Boise River (NFBR), Idaho, and some of its tributaries (Gott, 1998). Extensive and severe wildfires in summer, 1994, decreased soil permeability in coarse, granitic soils. During the following summer, convective rain storms caused widespread surface runoff and generated debris flows that delivered large volumes of sandy bed load to NFBR and its major tributaries. Although hydraulic data are lacking, these channels are relatively steep ($S > 0{\cdot}01$), have mostly planar streambeds (*sensu* Montgomery and Buffington, 1997) armoured with cobbles and boulders, and thus can be assumed to have high Froude numbers at formative flows. Initially, streambeds were locally buried by sand and fine gravel during relatively low flows. During the following winter and nival spring flood, these sediment waves were rapidly dispersed, and the wave front in the main channel advanced as far as 50 km to a reservoir. Neither leading nor trailing edges of the waves could be clearly discerned. Instead, a sandy mode filled surface interstices of re-exposed armour layers to varying degrees from the points of input to the reservoir. In summary, the waves evolved very rapidly and were essentially dispersive, as was observed in the corresponding experimental run.

As in the experiment, beds of pure sand and fine gravel in NFBR were exposed initially to tractive forces during flows that could have entrained some of the coarse, underlying armour had it been exposed. Fine material, some of which was suspended intermittently, was apparently rapidly stripped from these smooth beds and transported at high velocities downstream until it became hidden among exposed cobbles and boulders. The fine-grained waves were thereby dispersed rapidly and manifested as a secondary mode in varying concentrations among the armour layer.

A final example of sandy waves is provided by the East Fork River, Wyoming. Here, large volumes of sand are transported by irrigation-return flow in the Muddy River and deposited in the East Fork each summer when flows in the East Fork are low (Andrews, 1979). The following nival flood in the East Fork mobilizes these deposits. Thus, sandy waves are formed and transported annually in a gravel-bed river. These waves are significantly translational (Meade, 1985), although they soon disperse downstream (K. Prestegaard, unpublished data). The difference between the translational waves in the East Fork and the dispersive waves in NFBR and the experimental channels is that the Froude number in the East Fork is low ($F = 0{\cdot}36$) even at bankfull stage (Andrews, 1979). Therefore, this exception supports theoretical and experimental evidence: bed material waves in gravel-bed channels are predominantly dispersive unless the Froude number is low during transport events.

DISCUSSION AND CONCLUSIONS

We have focused our examination of bed material waves on those contained in gravel-bed channels that do not vary in width or gradient and do not adjust their width to sediment supply. Within this scope, wave evolution can be characterized as dispersive, except for limited cases of fine-grained waves in channels with $F \ll 1$. Small particle size of wave material can promote translation, but its primary influence apparently is to impart greater mobility to bed material and thereby accelerate wave evolution.

Factors other than those we have considered may enhance wave translation. Aggradation may become pronounced downstream, for example, as a wave front advances into a reach where a decrease in channel gradient causes the ratio of storage to transport resulting from an increase in sediment load to increase. Advancing zones of increased transport at the leading edge of a wave may activate sediment stored in unstable reaches and propagate a more pronounced wave downstream, as observed by Wathen and Hoey (1998) in Allt Dubhaig, Scotland. Such phenomena, as well as interchanges of sediment between channels and floodplains, need to be investigated in order to better understand the evolution of bed material waves in natural systems.

Results from recent modelling, experimentation and field studies described above are consistent with previous studies that describe characteristics of bed material waves and the channels that bear them (Table I).

Table I. Characteristics of previously described bed material waves

Wave	F/S [a]	Dominant particle sizes: wave/bed material	Evidence	Wave behaviour
East Fork River, Wyoming Meade, 1985	0·36/0·0007	Sand/gravel and sand	Cross-sections sounded during flood season	Sand waves created by annual tributary inputs translate downstream as they disperse.
Fall River, Colorado Pitlick, 1993	0·5/0·003	Sand/gravel	Cross-sections surveyed during wave migration	Sand wave advances rapidly downstream and fills channel upstream of constriction before dissipating.
Kowai River, New Zealand Beschta, 1983	NA [b]	Gravel/gravel	Cross-sections surveyed after peak aggradation event	The depth and volume of erosion of flood deposits decreased downstream.
Madison River, Montana Turner, 1995	NA/0·002	Gravel/gravel	Maps and surveys of terraces	Large, high, landslide-generated wave maintains stationary dam crest as it erodes and extends downstream.
Navarro River, California Hansler, 1998; Sutherland, 1998	?/0·0028	Gravel and sand/ gravel and sand	Annual topographic surveys starting immediately after wave emplacement	Landslide-generated wave is stationary as it disperses upstream and downstream. Much material lost to abrasion and suspension.
Mountain Creek, British Columbia Roberts and Church, 1986	NA/0·03	Gravel/gravel	Terrace and channel elevations over extrapolated bedrock exposures	Wedge deposits incise and stabilize upstream as aggradational front advances downstream.
North Fork Boise River, Idaho Gott, 1998	$\leq 1/>0{\cdot}01$	Sand/gravel	Large-scale aerial photographs; sand thickness	Sand advances rapidly downstream and thins but trailing edge remains stationary.
Redwood Creek, California Madej and Ozaki, 1996	Wide range	Gravel and sand/ gravel and sand	Annual cross-section surveys beginning 9 years after wave was formed	Locus of aggradation shifts downstream.
Ringarooma and George River, Tasmania Knighton, 1989, 1991	NA	Sand/gravel	Mass conservation modelling; limited bed-elevation sites; channel widths from aerial photographs	Sediment sources were diffuse and shifted downstream. Model predicts translation and dispersion of sand wave. Field data indicate downstream shift in aggradation.
Sacramento River tributaries, California Gilbert, 1917; Meade, 1982; James, 1993, 199	Wide range	Sand and gravel/sand and gravel	Incidence of increased flooding, terrace elevations	Incising wedges of mining debris in mountain channels shed selectively transported sand and silt that accumulated in low gradient rivers downstream. Much sediment remains in long-term storage.
South Fork Salmon River, Idaho Megahan *et al*., 1980; Bohn and Megahan, 1991	NA/<0·02	Sand and gravel/ gravel and sand	Periodic cross-section surveys following wave deposition	Degradation at each location was proportional to original (maximum) aggradation. No translation was evident.
South Tyne, England Macklin and Lewin, 1989	NA/0·004–0·013	Sand and gravel/gravel	Changes in width of active Gravel; metal concentrations	Aggradation mostly in five sedimentation zones; aggradation decreasing downstream; no translation of single wave.

[a] F = mean bankfull Froude number; S = channel gradient.
[b] Not available.

The behaviour of bed material waves remains uncertain in many cases because of the difficulty of reconstructing original channel conditions and early stages of wave evolution. Moreover, macroform-scale waves (equivalent in size to single bars) have received more attention than the megaform-scale waves (equivalent to bar assemblages) (Jackson, 1975; Church and Jones, 1982; Nicholas *et al.*, 1995). Clear examples of wave translation are thus less likely to be available. Nonetheless, unequivocal evidence for significant translation of megaform-scale, bed material waves in gravel-bed channels is restricted to a few cases consistent with our model predictions: low-amplitude, fine-grained waves moving through channels with low Froude number.

Despite the common occurrence of large, discrete inputs of sediment (e.g. landslides), our impression from searching for bed material waves to examine in the field or to review from the literature is that identifiable waves in natural channels are rare. The predominance of dispersion over translation and the dimensions of reported waves relative to their channels may help to explain why. Relative wave amplitude, measured as the ratio of mean height above the original streambed to wave length (H/L), is commonly low ($\leq 0{\cdot}001$) and decreases with age (Figure 4), suggesting that dispersion is strong, although waves such as those in the Ringarooma River (Knighton, 1989) and Redwood Creek (Madej and Osaki, 1996) originate from dispersed sources. Let us assume that, with no augmentations from new inputs or bank erosion, a dispersing wave becomes unrecognizable when its mean amplitude decreases to some fraction of bankfull depth (kd_{BF}). The mean amplitude of most reported waves falls within the range of $0{\cdot}2d_{\mathrm{BF}} < \mathrm{H} < d_{\mathrm{BF}}$ (Figure 5). Wave length measured in channel widths (w) when it 'disappears' is a function of wave volume (V) measured in unit channel volumes ($d_{\mathrm{BF}}w^2$) (Figure 5). The criterion for wave visibility is simply $L < V/k$, where $k = H/d_{\mathrm{BF}}$ is the threshold of visibility. For example, assuming no particle abrasion and $k = 0{\cdot}2$, a wave that initially filled a channel to its banks over a reach of five channel widths would disappear after it dispersed over 25 channel widths.

Put simply, a bed material wave must be large or young to be readily discerned. The upper limit of relative amplitudes likely to be found in natural waves is also plotted in Figure 5 (shaded line, $H/L = 0{\cdot}001$); the width of the line denotes a field of values for $10 < w/d_{\mathrm{BF}} < 50$. New waves are likely to plot near the limit of relative amplitudes and then disperse toward the limit of visibility. Values for observed waves mostly plot between these limits and indicate that recognizable waves have a minimum volume of at least several unit

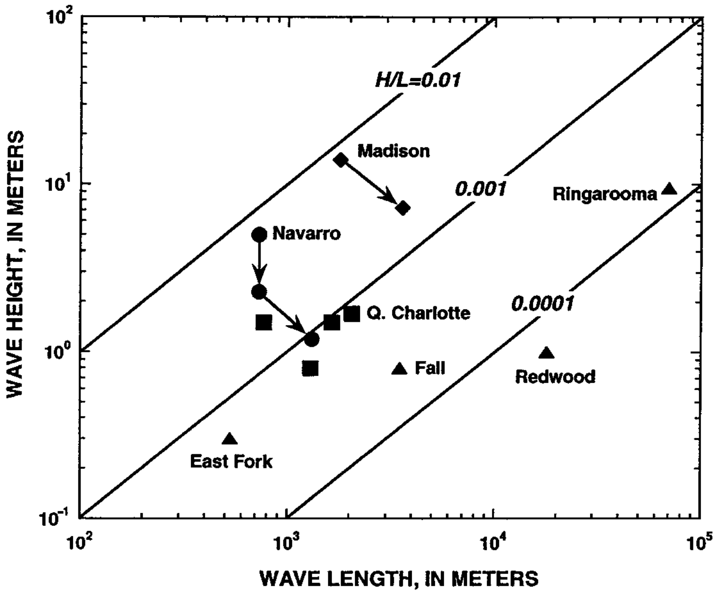

Figure 4. Mean wave height versus wave length. Arrows join stages of evolution given in number of years since sediment input. References: East Fork River, Meade (1985); Fall River, Pitlick (1993); Madison River, Turner (1995); Navarro, Sutherland *et al.* (1998); Queen Charlotte Island, Roberts and Church (1986); Redwood Creek, Madej and Ozaki (1996); Ringarooma River, Knighton (1989)

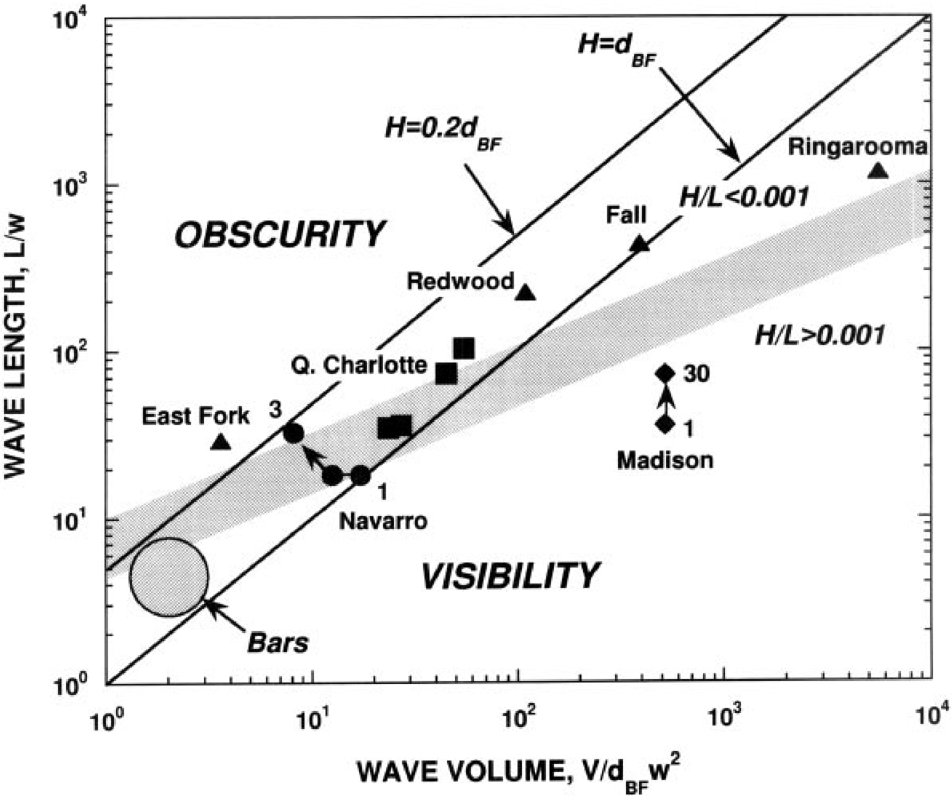

Figure 5. Sediment wave length versus wave volume. Heavy lines represent values corresponding to wave height in multiples of bankfull depth. The shaded line represents values at $H/L = 0{\cdot}001$ for $10 < w/d_{BF} < 50$

channel volumes. (As a point of reference, we estimate that alternate bars (macroforms) have a characteristic volume of 1·5 unit channel volumes.)

The dominance of dispersion suggests that large sediment inputs can be routed as dispersive bed-material waves advancing at the rate of some characteristic particle velocity and losing mass to abrasion and suspension. At larger scales, an assumption of zero wave translation could simplify sediment routing in drainage basins, where it may not be appropriate to use sediment transport equations to model fluxes through sediment storage reservoirs. Finally, dispersion of sediment waves may underlie the commonly observed low bed load transport rates in gravel-bed rivers, as fluctuations in transport rate associated with large point sources would tend to be quickly attenuated as the bed-material waves dispersed.

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APPENDIX A: DERIVATION OF EQUATION 6

It can be shown that Equation 6 is identical to Equation 13 derived in Lisle *et al*. (1997). Equation 6 is presented because it separates terms associated with dispersion from those associated with translation. A version of the MPM equation is used for the sediment-transport function in Equation 5,

$$q_s = \frac{K\left(c_f u^2\right)^{3/2}}{g R_s} \tag{A1}$$

and written in terms of the Froude number using Equation 4.

$$q_s = \frac{K c_f^{3/2} q F^2}{R_s} \tag{A2}$$

Replacing this with q_s in Equation 3,

$$\frac{\partial n}{\partial t} = \frac{-K c_f^{3/2}}{R_s(1-p)} \frac{\partial q F^2}{\partial x} \tag{A3}$$

assuming c_f is constant. Equations 1 and 2 can be combined and solved for F^2 by approximating the friction slope $S_f = \frac{\tau}{\rho g h}$ by using the Chezy equation $u = C\,(hS_f)^{1/2}$.

Substituting this into Equation A3 and rearranging results gives

$$\frac{\partial n}{\partial t} = \frac{K c_f^{1/2}}{R_s(1-p)} \left[\frac{\partial}{\partial x} \left(q \left\langle \frac{\partial n}{\partial x} + \left(1 - F^2\right) \left(\frac{\partial h}{\partial x}\right) \right\rangle - q \left\langle \frac{u}{gh} \frac{\partial h}{\partial t} - \frac{1}{g} \frac{\partial u}{\partial t} \right\rangle \right) \right] \tag{A4}$$

and, disregarding unsteady-flow terms, this becomes Equation 6

$$\frac{\partial n}{\partial t} = \frac{K q c_f^{1/2}}{R_s(1-p)} \left[\frac{\partial n^2}{\partial x^2} + \left(\frac{\partial}{\partial x} \left(1 - F^2\right) \frac{\partial h}{\partial x} \right) + \cdots \right]$$

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